THEORETICAL ANALYSIS OF OPEN RING LINE

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Abstract

The open ring line that presents low losses is studied theoretically by mean of Hankel transform. Electromagnetic field, stored energy, power flow and dispersion relation are calculated. Measurements have perfectly corroborated theoretical results.

Introduction

The study of low loss transmission lines has been developed in the course of the last years[1-2] owing to possible practical applications : railway traffic control, railway obstacle detection and telecommunications. The ring line described in reference[3] presents losses less than 5 db/km in L band. This paper presents the theoretical analysis of this line consisting of equally spaced metallic rings (figure 1).

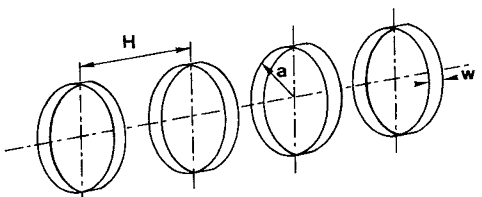

Fig. 1 : Open ring line (2a = 7.05 cm, H = 3 cm, w = 0.5 cm)

In order to simplify the analysis, we assumed the rings to be infinitely thin perfectly conducting tapes whose width w is small compared with the line pitch H. The ring line whose axis is taken to coïncide with the z axis is assumed to be infinite in extent. The fields are produced by azimuthal currents which only flow along the rings. In the cylindrical coordinate system (r, θ, z) the current densities can be written in the form :

$$J_\theta = j_\theta \cos n\theta \, e^{j\omega t} = A\,\delta(r-a)e^{j\omega t-\beta z}\cos n\theta \sum_{p=-\infty}^{\infty} f_p(z) \qquad (1)$$

where A is a constant which determines current and field intensity.

n is a positive integer which characterizes the symmetry in θ of currents and fields.

βH is the phase shift between two successive rings.

$f_p(z)e^{-j\beta z}$ is the current distribution across the $p^{\text{ième}}$ ring.

A reasonable assumption for this distribution is that the variation of current density across w approximates that on an isolated narrow thin ring :

$$f_p(z) = \begin{cases} \dfrac{w}{\sqrt{(z+\frac{w}{2}-pH)(pH+\frac{w}{2}-z)}} & |z-pH|<\frac{w}{2} \\ 0 & \text{elsewhere} \end{cases} \qquad (2)$$

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This approach was given by Sensiper in his helical line study[4].

The variation in cos nθ for the current density determines the azimuthal variation of field components :

$$\begin{aligned} E_r &= e_r(r,z)\sin n\theta\, e^{j\omega t} \\ E_\theta &= e_\theta(r,z)\cos n\theta\, e^{j\omega t} \\ E_z &= e_z(r,z)\sin n\theta\, e^{j\omega t} \\ H_r &= h_r(r,z)\cos n\theta\, e^{j\omega t} \\ H_\theta &= h_\theta(r,z)\sin n\theta\, e^{j\omega t} \\ H_z &= h_r(r,z)\cos n\theta\, e^{j\omega t} \end{aligned} \qquad (3)$$

Hankel transform

The n order Hankel transformation[5-6] transforms a fonction u(r) into a new function $u^{\{n\}}(s)$ such as

$$u^{\{n\}}(s) = \int_0^\infty rJ_n(rs)u(r)\,dr \qquad u(r) = \int_0^\infty sJ_n(rs)u^{\{n\}}(s)\,ds \qquad (4)$$

Operationnal properties :

$$\begin{aligned} \left[\frac{u}{r}\right]^{\{n\}} &= \frac{s}{2n}\left[u^{\{n+1\}} + u^{\{n-1\}}\right] \\ \left[\frac{du}{dr}\right]^{\{n\}} &= -\frac{s}{2n}\left[(n+1)\,u^{\{n-1\}} - (n-1)\,u^{\{n+1\}}\right] \\ \left[\frac{1}{r}\frac{d(ru)}{dr}\right]^{\{n\}} &= \frac{s}{2}\left[u^{\{n+1\}} - u^{\{n-1\}}\right] \\ \left[\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{n^2}{r^2}u\right]^{\{n\}} &= -s^2\,u^{\{n\}} \end{aligned} \qquad (5)$$

and integral product :

$$\int_0^\infty s\,u^{\{n\}}(s)\,v^{\{n\}}(s)\,ds = \int_0^\infty r\,u(r)\,v(r)\,dr \qquad (6)$$

are well suited for cylindrical coordinate system.

Field component expressions

From Maxwell's equations,we deduced the following relations satisfied by the longitudinal components :

$$\begin{aligned} \frac{\partial^2 e_z}{\partial r^2} + \frac{1}{r}\frac{\partial e_z}{\partial r} - \frac{n^2}{r^2}e_z + \frac{\partial^2 e_z}{\partial z^2} + \frac{\omega^2}{c^2}e_z &= -\frac{jn}{\omega\varepsilon r}\frac{\partial j_\theta}{\partial z} \\ \frac{\partial^2 h_z}{\partial r^2} + \frac{1}{r}\frac{\partial h_z}{\partial r} - \frac{n^2}{r^2}h_z + \frac{\partial^2 h_z}{\partial z^2} + \frac{\omega^2}{c^2}h_z &= -\frac{1}{r}\frac{\partial(rj_\theta)}{\partial r} \end{aligned} \qquad (7)$$

Introducing the wavenumber k = ω/c and using Hankel transform we obtain :

$$\frac{\partial^2 e_z^{\{n\}}}{\partial z^2} + (k^2-s^2)\,e_z^{\{n\}} = -\frac{jn}{\omega\varepsilon}\frac{\partial}{\partial z}\int_0^\infty J_n(rs)\; j_\theta\; dr$$

$$\frac{\partial^2 h_z^{\{n\}}}{\partial z^2} + (k^2-s^2)\,h_z^{\{n\}} = s\int_0^\infty r\, J'_n(rs)\; j_\theta\; dr \qquad (8)$$

In the current density expression (1), the periodic function $\sum_p f_p(z)$ given by (2) is developed in Fourier series:

$$j_\theta = A\,\delta(r-a)\,e^{-j\beta z}\sum_{p=-\infty}^{\infty} f_p(z) = I\,\delta(r-a)\sum_{m=-\infty}^{\infty} D_m\, e^{-j\beta_m z}$$

where $I = A\,w$ is the intensity of the current $I\cos n\theta\,\exp j(\omega t-\beta z)$ through the ring

$$D_m = \frac{1}{\pi}\int_{-H/2}^{+H/2}\frac{e^{2\pi jmz/H}}{\sqrt{(z+\frac{w}{2})(\frac{w}{2}-z)}} = J_o(m\pi\,\frac{w}{H})$$

$$\beta_m = \beta + 2m\pi/H$$

$$\alpha_m = \sqrt{\beta_m^2 - k^2}$$

then we have :

$$e_z^{\{n\}} = \frac{nI}{\omega\varepsilon H}\sum_{m=-\infty}^{\infty}\beta_m D_m\, e^{-j\beta_m z}\,\frac{J_n(as)}{s^2+\alpha_m^2}$$

$$h_z^{\{n\}} = -\frac{Ia}{H}\sum_{m=-\infty}^{\infty} D_m\, e^{-j\beta_m z}\,\frac{s\,J'_n(as)}{s^2+\alpha_m^2} \qquad (9)$$

Transverse components are deduced from longitudinal components by mean of (4) and (5)

$$(h_r+h_\theta)^{\{n+1\}} = \frac{1}{s}\left[j\omega\varepsilon e_z^{\{n\}} - \frac{\partial h_z^{\{n\}}}{\partial z}\right]$$

$$(h_r-h_\theta)^{\{n-1\}} = \frac{1}{s}\left[j\omega\varepsilon e_z^{\{n\}} + \frac{\partial h_z^{\{n\}}}{\partial z}\right]$$

$$(e_r-e_\theta)^{\{n+1\}} = \frac{1}{s}\left[j\omega\mu h_z^{\{n\}} - \frac{\partial e_z^{\{n\}}}{\partial z} - \frac{jn}{\omega\varepsilon}\int_0^\infty J_n(rs)\,j_\theta\,ds\right] \qquad (10)$$

$$(e_r+e_\theta)^{\{n-1\}} = \frac{1}{s}\left[j\omega\mu h_z^{\{n\}} + \frac{\partial e_z^{\{n\}}}{\partial z} + \frac{jn}{\omega\varepsilon}\int_0^\infty J_n(rs)\,j_\theta\,ds\right]$$

Field components are obtained by mean of inverse Hankel transform (4) and using the following integral[4] :

$$F_{nm}(a,r) = \int_0^\infty s\,\frac{J_n(as)J_n(rs)\,ds}{s^2+\alpha_m^2} = \begin{cases} K_n(\alpha_m a)\, I_n(\alpha_m r) & r<a \\ I_n(\alpha_m a)\, K_n(\alpha_m r) & r>a \end{cases} \qquad (11)$$

Field component expressions are ($e^{j\omega t}$ is omitted)

$$E_z = \frac{nI}{\omega\varepsilon H}\sin n\theta\sum_{m=-\infty}^{\infty}\beta_m D_m F_{nm}\, e^{-j\beta_m z}$$

$$E_r = \frac{jnI}{\omega\varepsilon H}\sin n\theta\sum_{m=-\infty}^{\infty}\frac{D_m}{\alpha_m^2}\left[\beta_m^2\frac{\partial F_{nm}}{\partial r} + \frac{k^2 a}{r}\frac{\partial F_{nm}}{\partial a}\right]e^{-j\beta_m z}$$

$$E_\theta = \frac{jI}{\omega\varepsilon H}\cos n\theta\sum_{m=-\infty}^{\infty}\frac{D_m}{\alpha_m^2}\left[\frac{n^2}{r}\beta_m^2 F_{nm} + k^2 a\,\frac{\partial^2 F_{nm}}{\partial a\,\partial r}\right]e^{-j\beta_m z}$$

$$H_z = \frac{-Ia}{H}\cos n\theta\sum_{m=-\infty}^{\infty} D_m\frac{\partial F_{nm}}{\partial a}\, e^{-j\beta_m z} \qquad (12)$$

$$H_r = \frac{-jI}{H}\cos n\theta\sum_{m=-\infty}^{\infty}\frac{D_m}{\alpha_m^2}\beta_m\left[\frac{n^2}{r}F_{nm} + a\,\frac{\partial^2 F_{nm}}{\partial a\,\partial r}\right]e^{-j\beta_m z}$$

$$H_\theta = \frac{jnI}{H}\sin n\theta\sum_{m=-\infty}^{\infty}\frac{D_m}{\alpha_m^2}\beta_m\left[\frac{\partial F_{nm}}{\partial r} + \frac{a}{r}\frac{\partial F_{nm}}{\partial a}\right]e^{-j\beta_m z}$$

Stored energy, power flow and dispersion relation

Expressions of :

time average magnetic stored energy per unit length

$$\bar{W}_M = \frac{\pi\mu}{4}\int_0^\infty (h_r h_r^* + h_\theta h_\theta^* + h_z h_z^*)\,r\,dr$$

time average electric stored energy per unit length

$$\bar{W}_E = \frac{\pi\varepsilon}{4}\int_0^\infty (e_r e_r^* + e_\theta e_\theta^* + e_z e_z^*)\,r\,dr$$

time average power flow

$$P_z = \frac{\pi}{4}\int_0^\infty\left[(e_r h_\theta^* - e_\theta h_r^*) + (e_r^* h_\theta - e_\theta^* h_r)\right]r\,dr$$

are transformed using the following identities

Dispersion relation	$\sum_{m=-\infty}^{\infty}\left[n^2\frac{\beta_m^2}{\alpha_m^2} I_n(\alpha_m a)\, K_n(\alpha_m a) + k^2 a^2\, I'_n(\alpha_m a)\, K'_n(\alpha_m a)\right] D_m^2 = 0$
	$\bar{W}_E = \frac{\pi I^2}{4\omega^2\varepsilon H^2}\sum_{m=-\infty}^{\infty}\left[\frac{k^2 a}{2\alpha_m}\left[\frac{n^2}{\alpha_m^2}(\beta_m^2+k^2)+k^2a^2\right]\left[I'_n K_n + I_n K'_n\right] - \frac{n^2\beta_m^2}{\alpha_m^4}(k^2-\alpha_m^2)\, I_n K_n - \frac{k^4 a^2}{\alpha_m^2} I'_n K'_n\right] D_m^2$
	$\bar{W}_M = \frac{\pi I^2 k^2}{4\omega^2\varepsilon H^2}\sum_{m=-\infty}^{\infty}\left[\frac{a}{2\alpha_m}\left[\frac{n^2}{\alpha_m^2}(\beta_m^2+k^2)+k^2a^2\right]\left[I'_n K_n + I_n K'_n\right] - \frac{n^2\beta_m^2}{\alpha_m^4} I_n K_n - \frac{\beta_m^2 a^2}{\alpha_m^2} I'_n K'_n\right] D_m^2$
	$P_z = \frac{\pi I^2}{2\omega\varepsilon H^2}\sum_{m=-\infty}^{\infty}\frac{\beta_m}{\alpha_m}\left[\frac{a}{2}\left[\frac{n^2}{\alpha_m^2}(\beta_m^2+k^2)+k^2a^2\right]\left[I'_n K_n + I_n K'_n\right] - \frac{n^2k^2}{\alpha_m^3} I_n K_n - \frac{k^2a^2}{\alpha_m} I'_n K'_n\right] D_m^2$

TABLE I - Dispersion relation, stored energy and power flow (argument of modified Bessel functions is $\alpha_m a$).

$$2(a_r a_r^* + a_\theta a_\theta^*) = (a_r+a_\theta)(a_r^*+a_\theta^*) + (a_r-a_\theta)(a_r^*-a_\theta^*)$$

$$2(a_r b_\theta^* - a_\theta b_r^*) = (a_r-a_\theta)(b_r^*+b_\theta^*) - (a_r+a_\theta)(b_r^*-b_\theta^*)$$

Then, $\overline{W}_M$, $\overline{W}_E$ and P_z are calculated using (6), (10), (9).

Dispersion relation is obtained by writting the equality $\overline{W}_E = \overline{W}_M$.

The same expression could be obtained by writting $\int_V \mathbf{E}.\mathbf{J}^* dV = 0$ because Maxwell's equation give the relation

$$\overline{W}_N - \overline{W}_E = \frac{j}{4\omega} \int_V \mathbf{E}.\mathbf{J}^* dV$$

Table I gives expression of dispersion relation, W_E, W_M and P_z. Total stored energy per unit length is:

$$\overline{W} = \overline{W}_E + \overline{W}_M = 2\,\overline{W}_M = 2\,\overline{W}_E.$$

The dispersion relation gives the dispersion curves satisfied by the different propagating modes of the ring line. The analysis of this expression shows that only one hybrid wave can propagate along the structure for each value of $n \neq o$ and the cylindrical symmetry mode ($n = o$) does not exist. We see, that the fundamental mode has dipolar symmetry. Figure 2 shows the dispersion curve of this mode.

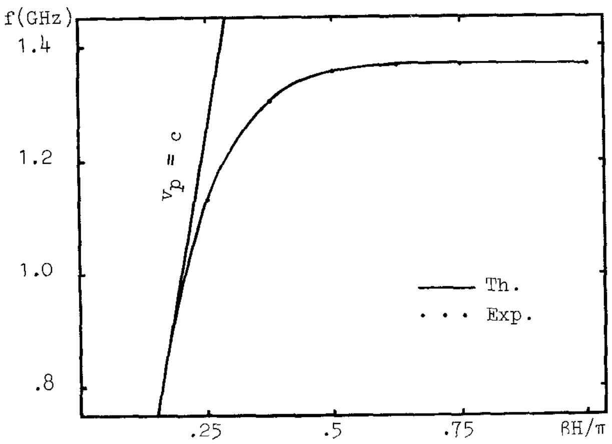

Figure 2 : Dispersion characteristic for the fundamental mode. Geometrical parameters : 2 a = 7.05 cm, H = 3 cm, w = .5 cm.

Experimental points are obtained with a section of this structure introduced between two metallic planes to obtain a resonator. The rings are of an aluminium alloy. Their geometrical parameters are equal to those of figure 1 but ring thickness is 0.05 cm. Two teflon supports hold up the rings and keep constant the period. The experimental curve is practically identical to the theoretical one (error is less than 0.7 %). Figure 3 showsthe variation of the stored energy and the power flow against the phase shift βH of the fundamental mode. Looking at figures 2 and 3, it can be noticed that group velocity $v_g = d\omega/d\beta$ measured on dispersion curve is equal to energy velocity $P_z/\overline{W}$.

Conclusion

This theoretical study provides detailed knowledge on the propagation of electromagnetic waves in the ring line. The Hankel transform allowed us to determine easely field components, stored energy and power flow. These expressions bring out the electromagnetic properties of lossless open ring line. Losses and electromagnetic energy distribution around the line will be studied in further papers.

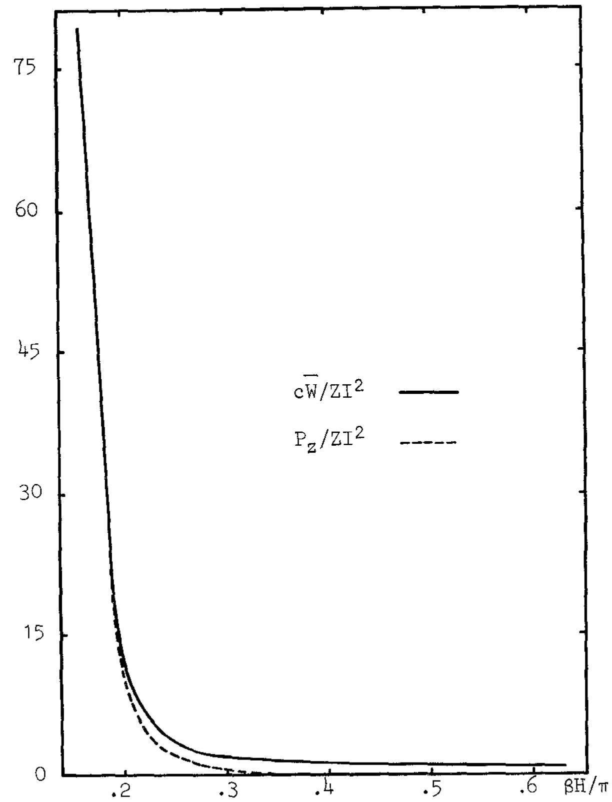

Figure 3 : Theoretical power flow and theoretical stored energy versus phase shift for the fundamental mode. $c = \sqrt{\varepsilon\mu}$ is the light velocity and $Z = \sqrt{\mu/\varepsilon}$ the impedance of medium. Geometrical parameters : 2 a = 7.05 cm, H = 3 cm, w = .5 cm.

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